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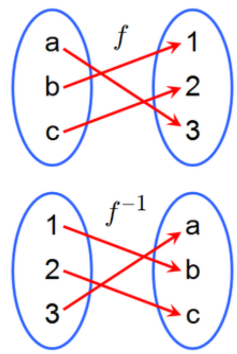
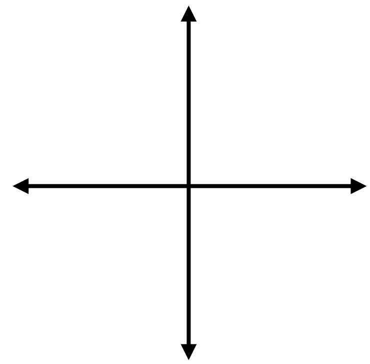
INVERSE





- An inverse function ____________________the original function.
- Formal definition: f and g are inverses if :

$$f(g(x)) = x \qquad g(f(x)) = x$$

- The **notation** for the inverse of $f(x)$ is ___________
- The domain of $f(x)$ is the _____________ of $f^{-1}(x)$
- The range of $f(x)$ is the _____________ of $f^{-1}(x)$
- Graphically the inverse is the reflection over the _______ line.

Step Box

1. $f(x) \rightarrow y$
2. Change x and y
3. Solve for y
4. New $y \rightarrow f^{-1}(x)$

COMPUTING THE INVERSE

EXAMPLE 1 Given: $f(x) = 3x + 2$

A) find $f^{-1}(x)$	B) Find the Domain of $f^{-1}(x)$

CHECKING INVERSES

Functions f and g are **inverses** only if $f(g(x)) = x$ and $g(f(x)) = x$

EXAMPLE 2 Are the following functions inverses of one another?

A) Given $f(x) = 3x + 2$ and $g(x) = \frac{1}{3}x - \frac{2}{3}$	**B)** Given $f(x) = \sqrt{x-2}$ and $g(x) = x^2 - 2$

PRACTICE: INVERSES

PROBLEM: Compute the inverse.

Step Box
1. $f(x) \to y$
2. Change x and y
3. Solve for y
4. New $y \to f^{-1}(x)$

1. Given $f(x) = \sqrt{x-5}$, find $f^{-1}(x)$

A	$f^{-1}(x) = \sqrt{x+5}$	B	$f^{-1}(x) = x^2 - 5$
C	$f^{-1}(x) = x^2 + 5$	D	$f(x) = \sqrt{x-5}$

2. Given $(x) = \frac{1}{x-4}$, find $f^{-1}(x)$

Step Box
1. $f(x) \to y$
2. Change x and y
3. Solve for y
4. New $y \to f^{-1}(x)$

A	$f^{-1}(x) = \dfrac{1}{x-4}$	B	$f^{-1}(x) = x + 4$
C	$f^{-1}(x) = \dfrac{1+4x}{x}$	D	$f^{-1}(x) = \dfrac{x}{x-4}$

PROBLEM: Check if the following functions are inverses.

3. $f(x) = 3x + 2 \quad g(x) = \frac{x-2}{3}$

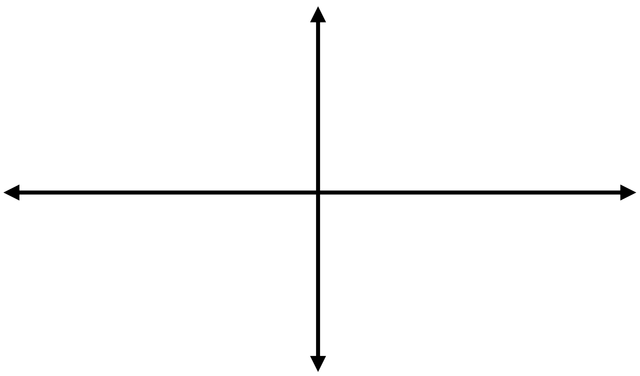
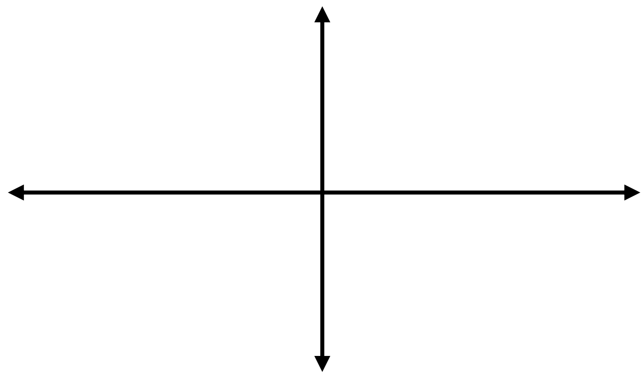
A	Yes	B	No

4. $f(x) = \sqrt[3]{x+7} \quad g(x) = x^3 + 7$

A	Yes	B	No

EXPONENTIAL FUNCTIONS

- An exponential function in the written in the form ______________ .
- The base is always a (**constant / variable**) and the exponent is always a (**constant / variable**)
- The value of the constant will determine the **shape** of the graph

Exponential Growth	Exponential Decay
$\boldsymbol{b} > 1$	$0 < \boldsymbol{b} < 1$
	

RULES OF EXPONENTS

Multiplication	Division	Raising to a Power	Negative Exponents
$\boldsymbol{x^a \cdot x^b = x^{a+b}}$	$\boldsymbol{\frac{x^a}{x^b} = x^{a-b}}$	$\boldsymbol{(x^a)^b = x^{a \cdot b}}$	$\boldsymbol{x^{-a} = \frac{1}{x^a}}$
EX:	**EX:**	**EX:**	**EX:**

EXAMPLE 1:

A) $\frac{3x^3y^5}{12x^2y^2}$

B) $\frac{x^3}{20y^2z} \cdot \frac{3x^5y^2}{y^3}$

C) $\frac{y^5}{14x^2y^2z} \div \frac{x^3}{7x^7y^2z}$

PRACTICE: EXPONENTS

PROBLEM: Simplify the following.

$x^a \cdot x^b = x^{a+b}$	$\frac{x^a}{x^b} = x^{a-b}$	$(x^a)^b = x^{a \cdot b}$	$x^{-a} = \frac{1}{x^a}$

1. $\frac{2x^3}{5y^7} \cdot \frac{10xy^2}{x^2}$

A	$\frac{20x^2}{5y^5}$
B	$4x^5y^2$
C	$50x^2y^5$
D	$\frac{4x^2}{y^5}$

2. $\frac{2x}{y^3} \div \frac{y^2}{x^2}$

A	$\frac{2x^3}{y^5}$
B	$\frac{2x}{y}$
C	$\frac{2xy^2}{x^2y^3}$
D	$\frac{3x^2}{y}$

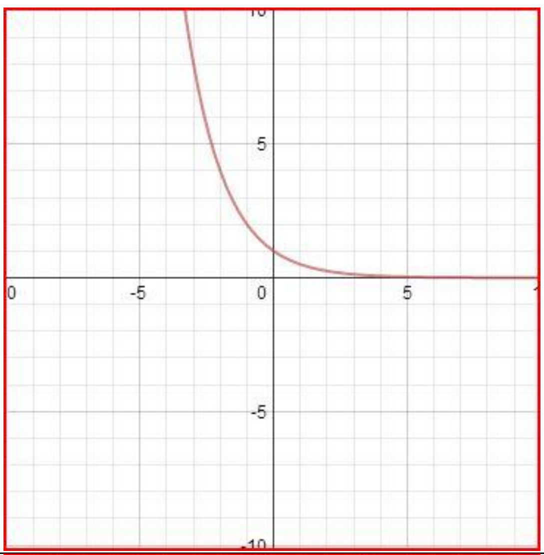
3. $\frac{3\sqrt{x}}{xy^7} \div \frac{21y^2}{x^2}$

A	$\frac{x^{2/3}}{7y^9}$
B	$\frac{7}{y^9}$
C	$\frac{x^{3/2}}{7y^9}$
D	$\frac{21x^2}{y^9}$

PROBLEM: Graph the following exponential function

4. $f(x) = -(2)^x$

A



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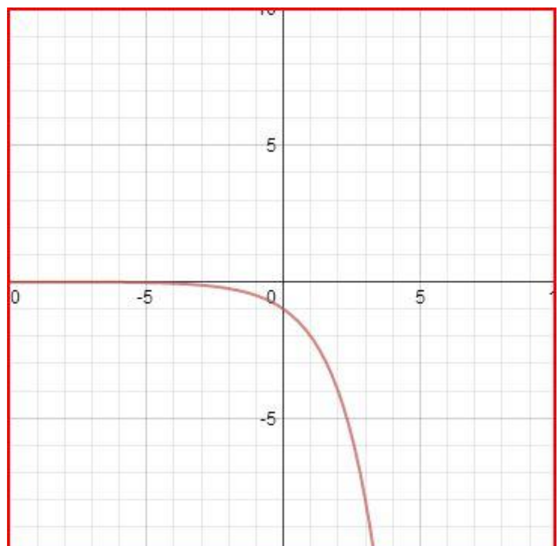

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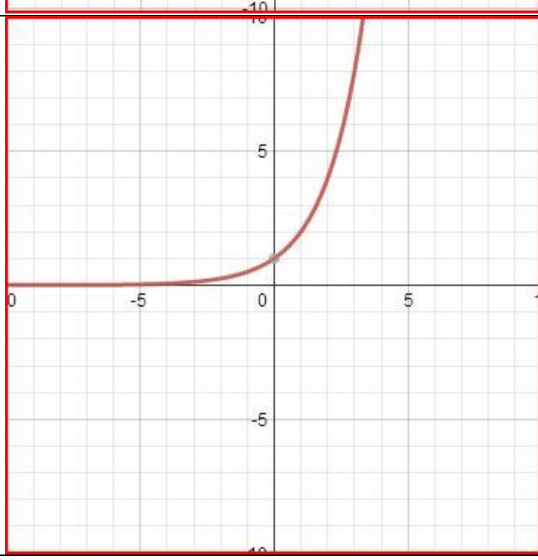

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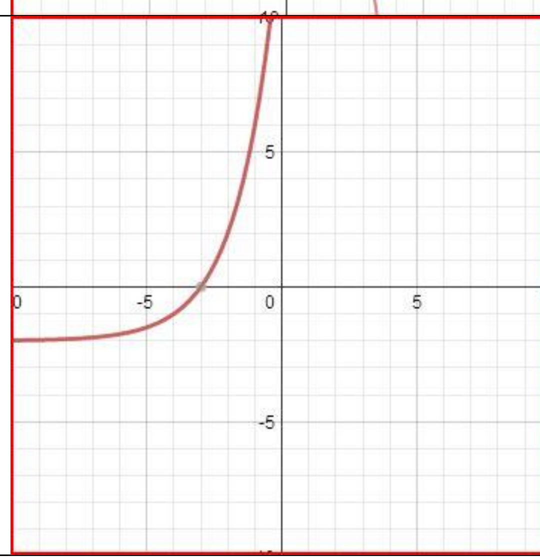
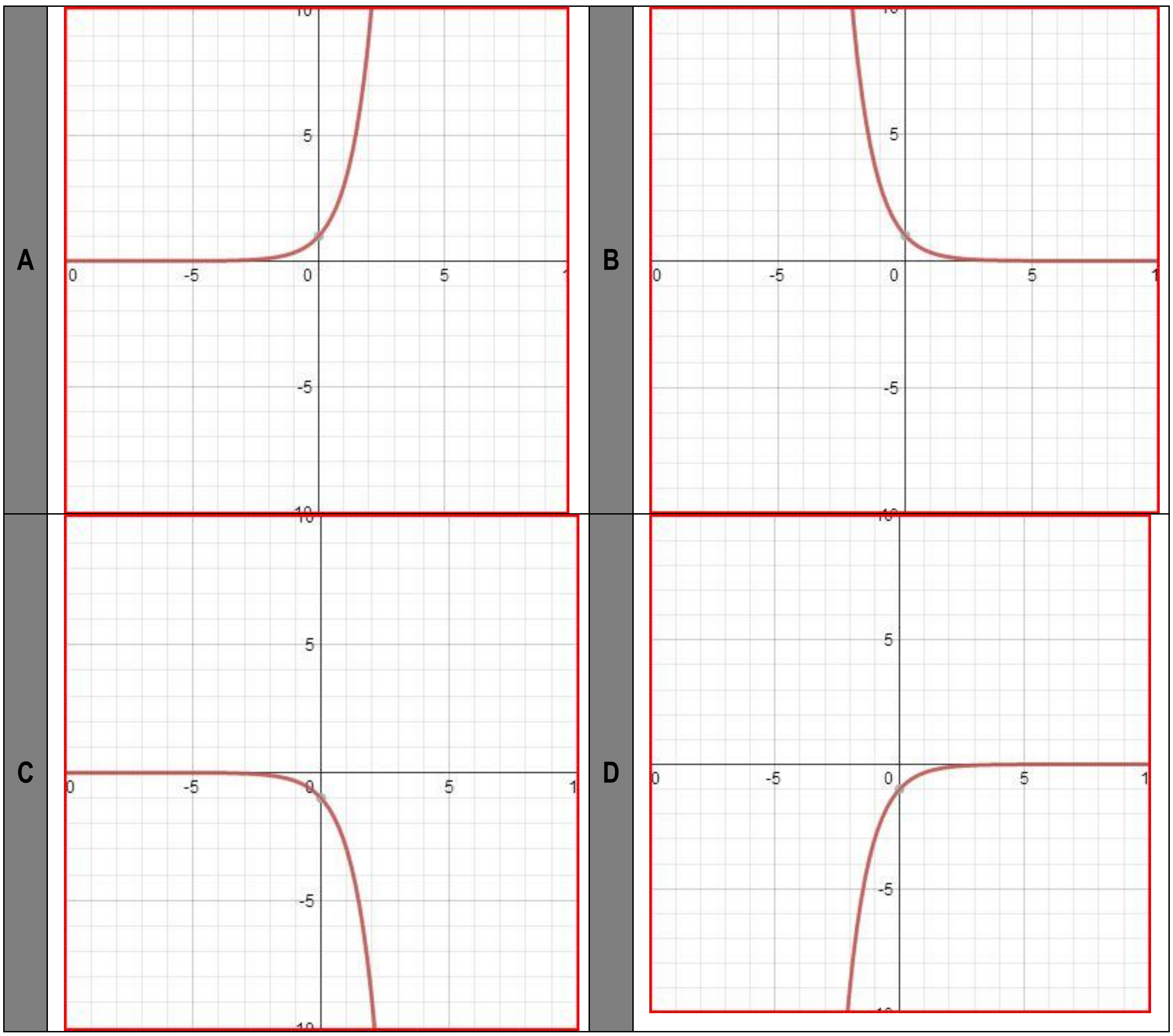


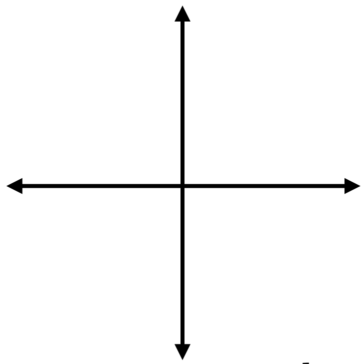
PROBLEM: Graph the following exponential function

5. $f(x) = \left(\frac{1}{3}\right)^x$



LOGARITHMIC FUNCTIONS

- A logarithm is an________________, the power to which a number must be __________ order to get some other number
- You always solve logarithms in **exponential** form.
- Logarithms and Exponentials are _________________



X	Y

X	Y

$$\log_b(x) = y \leftrightarrow b^y = x$$

EXAMPLE 1: Find the exact value of the expression.

A) $\log_3 9$	B) $\log_3 1$

PROPERTIES OF LOGARITHMS

Product Property	**Quotient Property**	**Power Property**	**Reciprocal Property**
$\log_b(x \cdot y) = \log_b x + \log_b y$	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$	$\log_b(x)^y = y\log_b x$	$\log_b\left(\frac{1}{x}\right) = -\log_b x$
EX: $\log_2(3x)$	EX: $\log_2\left(\frac{5}{x}\right)$	EX: $\log_2(5x)^3$	EX: $\log_3\left(\frac{1}{2}\right)$

EXAMPLE 2: Expand $\log_2 \frac{3(x+1)^2}{y}$

EXAMPLE 3: Compress $2\log_5(x) - \log_5(y)$

PRACTICE: LOGARITHMS

$\log_b(x \cdot y) = \log_b x + \log_b y$	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$	$\log_b(x)^y = y\log_b x$	$\log_b\left(\frac{1}{x}\right) = -\log_b x$	$x^{a/b} = \sqrt[b]{x^a}$

PROBLEM: Expand the following logarithm.

1. $\log_3 \frac{7x^2}{y}$

A	$\log_3 7 + 2\log_3 x + \log_3 y$
B	$\log_3 7 + 2\log_3 x - \log_3 y$
C	$\log_3 7 + \log_3 2x - \log_3 y$
D	$\log_3 14 - 2\log_3 x + \log_3 y$

2. $\log_5 \frac{(x+1)^3}{y}$

A	$\log_5 x + \log_5 1 + \log_5 y$
B	$\log_5 x + \log_5 1 - \log_5 y$
C	$3\log_5(x+1) + \log_5 y$
D	$3\log_5(x+1) - \log_5 y$

3. $\log_2 \frac{\sqrt{x}}{(2y-1)^3}$

A	$\log_2 x + \log_2(2y-1)$
B	$\log_2 x - 3\log_2(2y-1)$
C	$\frac{1}{2}\log_2 x + 3\log_2(2y-1)$
D	$\frac{1}{2}\log_2 x - 3\log_2(2y-1)$

PROBLEM: Compress the following logarithm.

$\log_b(x \cdot y) = \log_b x + \log_b y$	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$	$\log_b(x)^y = y\log_b x$	$\log_b\left(\frac{1}{x}\right) = -\log_b x$	$x^{a/b} = \sqrt[b]{x^a}$

4. $\log_2 4 + \log_2(x)$

A	$\log_2(4x)$
B	$\log_2(4 + x)$
C	$\log_2(x^4)$
D	$\log_2\left(\frac{4}{x}\right)$

5. $\log_5 x - 2\log_5 y$

A	$\log_5(x - y)$
B	$\log_5\left(\frac{x}{y^2}\right)$
C	$\log_2(xy)$
D	$\log_5(2xy)$

6. $\log_2 x - 3\log_2 y + \frac{1}{2}\log_2 z$

A	$\log_2(x - 3y + z)$
B	$\log_2\left(\frac{x\sqrt{z}}{3y}\right)$
C	$\log_2\left(\frac{x\sqrt{z}}{y^3}\right)$
D	$\log_2\left(\frac{x}{3y\sqrt{z}}\right)$