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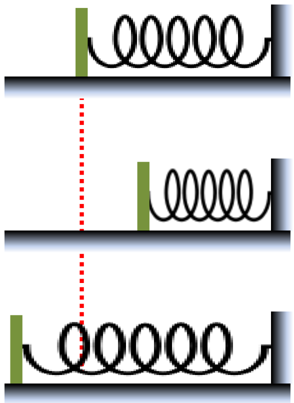
REVIEW SPRINGS

● When you push/pull against a spring with F_A, the spring pushes back (Newton's _____ Law):



- x = _________________ (________________ or _______________).
  - NOT the spring's length, but its **change** → x = ___________.
- k is the spring's __________________________ (Units: _________)
  - How _________ the spring is. Higher k → _________ to deform.
- F_S is a _________________ force, always opposite to deformation (____)
  - Always pulling spring back to its original length (x = ____).

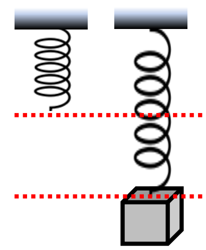
EXAMPLE 1: A 1.0 m-long spring is laid horizontally with one of its ends fixed. When you pull on it with 50 N, it stretches to 1.2 m. **(a)** What is the spring's force constant? **(b)** How much force is needed to compress it to 0.7 m?

● If you attach a mass to a vertical spring, and let the mass come down slowly:

- Its weight will stretch the spring, until they reach _________________:

  _______ = _______

- This also applies to a mass on top of a spring, slowly compressing it.

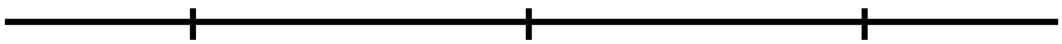


PRACTICE 1: A vertical spring is originally 60 cm long. When you attach a 5 kg object to it, the spring stretches to 70 cm. **(a)** Find the force constant on the spring. **(b)** You now attach an additional 10 kg to the spring. Find its new length.

A. PERIODIC MOTION: INTRO & DEFINITIONS

● When an object moves in repetitive way, we call this **P__________________**, aka **O______________.**

- The classic example of this motion is a mass connected to a spring, aka the mass-spring system:



x = ____	x = ____	x = ____
v = ____	v = ____	v = ____
a = ____	a = ____	a = ____
F = ____	F = ____	F = ____

● Definitions:

- Amplitude (A): Maximum displacement, **|x|** (always the __________________)
- Period (T): Time for one cycle (seconds/cycle) → From one point back to the same point! (not across)
- Frequency (f) = 1 / T (inverse of period, cycles/second). Angular Frequency → **w = ______ = ________.**

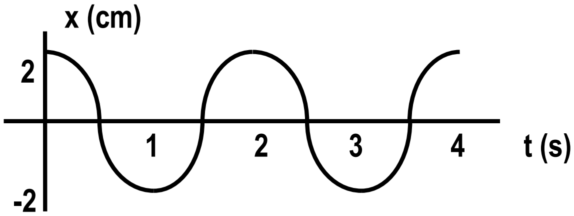
● As the system moves, the only force doing work on it is the ____________ force → ___ = ________.

● Condition for Simple Harmonic Motion (SHM):

The **R_________ F______ (__)** is DIRECTLY proportional to the ______________ from ______________ **(___).**

EXAMPLE A1: A mass on a spring is pulled 1m away from its equilibrium position, then released from rest. (a) Is this Periodic Motion? (b) Is this Simple Harmonic Motion (SHM)? The mass takes 2s to reach maximum displacement on the other side. Calculate the (c) amplitude, (d) period, (e) frequency, and (f) angular frequency of motion.

EXAMPLE A2**:** The graph shows position vs. time of a horizontal mass-spring system with k=400N/m. Find (a) amplitude, (b) period, (c) frequency, (d) angular frequency. (e) At what times does $\mathbf{v}_{max}$ occur? (f) At what times does $\mathbf{a}_{max}$ occur?



B. DESCRIBING MOTION / EQUATIONS

● Because the acceleration is NOT constant, we can NOT describe motion using Chapter 2 equations. **=(**

→ **So how?**

→ Find a(x):

ΣF = ma

x(t) = A cos(wt) **→ x_{MAX} =**

v(t) = –A w sin(wt) **→ v_{MAX} =**

a(t) = –A w^2 cos(wt) **→ a_{MAX} =**

=

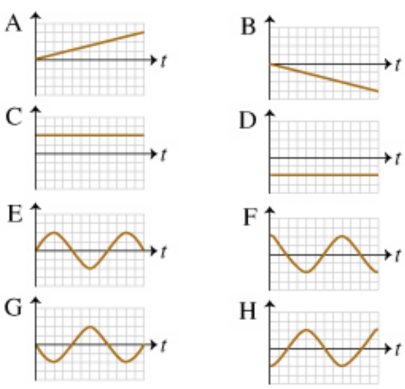
(Calculator in radians!)

→ Combine a(x) and a(t):

→ w =

(notice no A!)

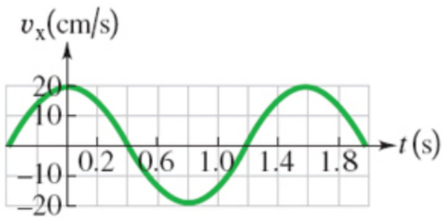
EXAMPLE B1: A mass-spring system is released from rest from the left (negative) of its equilibrium. Which of the following charts represents the system's (a) position over time, (b) velocity over time, and (c) acceleration over time?



EXAMPLE B3: A 4 kg mass is attached to a spring with force constant 200 N/m. The mass is then pulled 2 m and released from rest. Find the system's (a) amplitude, (b) angular frequency, and (c) velocity 0.5 s after being released.

PRACTICE B4: The graph below shows the velocity of a mass-spring system (mass M, spring k = 100 N/m) as it oscillates.

Find the system's (a) period, (b) frequency, (c) angular frequency, and (d) amplitude.
At what times is the system at its (e) equilibrium, (f) amplitude, (g) max speed, (h) max acceleration?
(i) Find the value of M.



PRACTICE B4: A 4 kg mass on a spring is released 5 m away from equilibrium position and takes 1.5 s to reach its equilibrium position. Find the spring's force constant.

PRACTICE B5: A 4-kg mass is attached to a vertical spring and oscillates at 2 Hz. If mass is moving with 10 m/s when it crosses its equilibrium position, find the system's (a) period, (b) amplitude, and (c) maximum acceleration.

EXAMPLE B6: When an object of mass M is attached to a spring of force constant k, the system oscillates horizontally with a period T. You then replace the object with a new object of mass 4 M. Find the new period, in terms of T.

PRACTICE B7: An object of mass M suspended by a spring vibrates with frequency f. When a second object is attached to the first, the system now vibrates with frequency f / 4. Calculate the mass of the second object, in terms of M.

C. ENERGY IN SIMPLE HARMONIC MOTION (S.H.M.)

● At each point of its motion, the mass-spring system can have two types of Energy: ____________________________.

- W_{NC} is ____, so total Mechanical Energy is _____________.

● Remember Energy problems are all about comparing energies at two points. Two special points here are:

- Equilibrium (x = ___) → **ME =**

- Amplitude (x = ___) → **ME =**

- Any point (x = ___) → **ME =**

- Comparing these points, we get:

$$U_A = K_{MID} = U_P + K_P \qquad v(x) = w\ \text{sqrt}(A^2 - x^2) \qquad v_{MAX} = +/- Aw\ (@\ x=0)$$

$$\tfrac{1}{2} kA^2 = \tfrac{1}{2} mV_{MAX}^2 = \tfrac{1}{2} kX_P^2 + \tfrac{1}{2} mV_P^2 \qquad a_{MAX} = +/- Aw^2\ (@\ x=A)$$

PRACTICE C1: You increase the amplitude of oscillation of a mass vibrating on a spring. Which statements are correct?

(a) Period of oscillation increases (b) Maximum acceleration increases (c) Maximum speed increases
(c) Max Kinetic Energy increases (d) Max Potential Energy increases (e) Max Total Energy increases

EXAMPLE C2: A 5 kg mass oscillates on a horizontal spring with amplitude of 0.1 m. If its max acceleration is 5 m/s², find its (a) max speed, (b) speed when it is at -0.04 m, and (c) the total mechanical energy of the system.

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D. VERTICAL MASS-SPRING SYSTEM

● Vertical mass-spring problems "work" almost exactly like horizontal ones.

- A key difference is that Amplitude is the ______________________ distance, not the _____________ distance.

EXAMPLE D1: You hang a 0.5m spring from the ceiling. When attach a 5kg mass to the spring, it stretches by 0.2m. Next, you pull the mass-spring system down an additional 0.3m and release it from rest. Find (a) the spring's force constant, (b) the spring's maximum deformation, and (c) the system's amplitude. (d) How long will it take for the mass to reach its maximum height for the first time after being released? (e) At its maximum height, how far from the ceiling is the block?

EXAMPLE D2: A chair of mass 30kg on top of a spring oscillates with a period of 2s. (a) Find the spring's force constant. You place an object on top of the chair, and it now oscillates with a period of 3s. (b) Find the object's mass.

E. SIMPLE PENDULUM

● Another example of Periodic motion is the Simple Pendulum:

- Restoring Force: ____________ → NOT directly proportional to displacement → NOT SHM!
- With small angle approximation, ___________, so restoring force ____________ →
- Similar to mass-spring, BUT: **x → ____. A → _______. w = ___________.**

● Find speed using Conservation of Energy:

→ h = ____________.

→ v_{MAX} = __________.

→ a_{MAX} = __________.

EXAMPLE E1: After landing on an unfamiliar planet, an astronaut constructs a simple pendulum of length 3m and mass 4kg. The astronaut releases the pendulum from 15 degrees with the vertical, and clocks one full cycle at 2s. (a) Calculate the acceleration due to gravity at the surface of this planet. (b) Find the max speed of the mass (where does this happen?).